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Blind Deconvolution and Separation Signal Processing via Inverse Model Approach

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Abstract *- The blind source deconvolution/separation problem is considered based on inverse deconvolution model application. A new approach to inverse model design based on unknown-input observer theory is considered with reference to blind separation problem. The method of reduced-order inverse model design in proposed and the conditions of inverse model parametric design problem solvability are found as a special type of observability. For special case of solvability conditions violation a regularized inverse model is proposed which ensures the possibility of arbitrary pole-placement of inverse model. The method of wave source signals identification and prediction is also proposed.*

Keywords *- blind separation, deconvolution, inverse models, nonminimum-phase systems, signal processing, unknown-input observer.*

I. INTRODUCTION

The blind source deconvolution/separation problem is to recover unknown independent source signals from sensors outputs, propagate through the dynamical mixing system, without any *a priory* knowledge of the original signals [1]. There are a lot of practical applications of blind source deconvolution/separation in many fields of data acquisition and signal processing, such as processes instrumentation and control, audio and acoustics, biomedical experiments. A number of methods for such a problem have been developed in last years; one of the most perspectives is a state-space approach, based of a suitable modification of Kalman Filter [2].

Blind deconvolution, treated as an input signals recovery, is closely related with the general problem of dynamic system inversion. Such a method is restricted by typically insufficient inverse system dynamic properties, because the parameters of inverse deconvolution model are strictly determined by the parameters of mixing system. For example, inverse model for nonminimum-phase mixing system obviously will be unstable.

In this paper a new approach to reduced-order inverse model design based on unknown-input observer (UIO) [3] theory is considered with reference to blind separation problem. The method of UIO-based inverse model of mixing system design is suggested and the conditions of inverse model parametric design problem solvability are examined. For special case of solvability conditions violation a regularized inverse model is proposed which ensures the possibility of arbitrary pole-placement of designed inverse model and source signal real-time identification and forecasting.

The proposed method of mixing system input signal recovery open up the possibilities of blind source signals identification and prediction. The ordinary prediction methods usually use the simple models like "trend + noise" or ARMA models in combination with recurrent parameters identification algorithms [4]. In practice, however, source signals have a more complex structure, for example, like non-periodic oscillating function (so-called wave signals). The identification problem has become more complicated when both amplitudes and frequencies are arbitrary and unknown and moreover changing in time. In general case such signals are non-periodic and unknown frequencies extraction by the DFF methods [5] in impossible. Alternative approach for wave signals prediction uses a tuning digital filters implementation [6]. The proposed identification method is based on special assignment of wave component auto-regression model as a superposition of harmonics with tuning amplitudes and frequencies. In such a case the suitable identification algorithm ensures non-stationary frequencies real-time tracking.

II. PROBLEM STATEMENT

Consider blind source deconvolution/separation problem with reference to multivariable discrete time state-space mixing system model:

$$x_{k+1} = Ax_k + Bu_k, \quad y_k = Cx_k, \tag{1}$$

where $x_k \in \mathbf{R}^n$ - mixing system state vector at step *k*, $u_k \in \mathbf{R}^p$ - vector of unknown input signals, $y_k \in \mathbf{R}^q$ - vector of output measured signals. Without loss of generality one can assume that the model matrices are of full rank, namely $\operatorname{rank} B = p$, $\operatorname{rank} C = q$.

Dynamic system

$$\begin{aligned} \bar{x}_{k+1} &= A^I \bar{x}_k + B_1^I y_k + B_2^I y_{k+1}, \\ \hat{u}_k &= C^I \bar{x}_k + D_1^I y_k + D_2^I y_{k+1}, \end{aligned} \tag{2}$$

with state vector $\hat{x}_k \in \mathbf{R}^{n-q}$ will be considered as a *reduced-order inverse model* of mixing system (1), if system (2) is asymptotically stable and the following conditions take place: $\|u_k - \hat{u}_k\| \to 0, k \to \infty$. Thus, $\hat{u}_k$ may be treated as one step delayed causal unknown input signal estimate, obtained by the inverse model (2).

The basic of proposed approach is the state-space representation of the mixing inverse model. If the mixing system invertibility conditions $S = CB \neq 0$ and $\operatorname{rank} S = q, q \geq p$ take place, the structure inversion algorithm may be applied. The inverse model design problem includes the matrices of system (2) determination. The suitable method must include the corresponding matrices parameterization and free tuning parameters selection from the simultaneous conditions of stability and desired dynamic properties. The most general way for such parameterization is the unknown-input observer (UIO) theory [3], and then the observer equation combined with the unknown input signal estimate may be treated as the designed inverse model.

III. INVERSE DECONVOLUTION MODEL

The minimal state-space realization of the inverse model may be obtained by means of reduced order UIO. Let $z = Rx_k \in \mathbf{R}^{n-q}$ be a vector of aggregated auxiliary variables, where $R \in \mathbf{R}^{n-q\times n}$ is the appropriate aggregate matrix, such that $\operatorname{rank}\left(C^{\mathrm{T}} \vdots R^{\mathrm{T}}\right) = n$, Then the state vector estimate $\hat{x}_k \in \mathbf{R}^n$ may be obtained by minimal-order UIO as follows:

$$\begin{aligned} \tilde{x}_{k+1} &= \bar{F}\tilde{x}_k + \bar{G}y_k, \\ \bar{x}_k &= \tilde{x}_k + \bar{H}y_k,\ \hat{x}_k = Py_k + Q\bar{x}_k, \end{aligned} \qquad (3)$$

where $\tilde{x}_k \in \mathbf{R}^{n-q}$ - observer state vector, and matrices $P_{n\times q}$ and $Q_{n\times n-q}$ are uniquely determined by selected aggregating matrix R and has the following properties:

$$\begin{pmatrix} P & Q \end{pmatrix} = \begin{pmatrix} C \\ R \end{pmatrix}^{-1}, \quad \begin{aligned} &CP = I_q, RQ = I_q, PC + QR = I_n, \\ &CQ = 0_{q\times n-q}, RP = 0_{n-q\times q}, \end{aligned} \qquad (4)$$

The observer design conditions (state estimation independence from unknown input) are:

$$\begin{aligned} &\left(R - \bar{H}C\right)A - \bar{F}\left(R - \bar{H}C\right) - \bar{G}C = 0, \\ &\left(R - \bar{H}C\right)B = 0. \end{aligned} \qquad (5)$$

and a corresponding solution of linear matrix equation (5) under the invertibility conditions fulfillment may be obtained as

$$\begin{aligned} &\bar{F} = R\Pi AQ, \quad \bar{G} = R\Pi A\left(H + P\Omega\right), \\ &\bar{H} = RBS^{+} = RH \end{aligned} \qquad (6)$$

where projection matrices

$$\Pi = I_n - BS^{+}C,\ \Omega = I_q - SS^{+},\ C\Pi = \Omega C, \qquad (7)$$

and "+" denotes Moore-Penrouze generalized inverse.

Taking an unknown input vector estimate as $\hat{u}_k = B^{+}\left(\hat{x}_{k+1} - A\hat{x}_k\right)$, the reduced-order inverse model equations may be obtained in the form:

$$\begin{aligned} \bar{x}_{k+1} &= \bar{F}\bar{x}_k + R\Pi APy_k + \bar{H}y_{k+1}, \\ \hat{u}_k &= B^{+}\left(H + P\Omega\right)\left[y_{k+1} - CAQ\bar{x}_k - CAPy_k\right], \end{aligned} \qquad (8)$$

As a result, the inverse model dynamics matrix $A^I = \bar{F} = R\Pi AQ$, depends from the arbitrary aggregating matrix R of given rank $n-q$, which may be considered as a tuning matrix.

It has been proved, that deviation vector $\bar{e}^x_k = Rx_k - \bar{x}_k$ and input signal estimation error $\bar{e}^u_k = u_k - \bar{u}_k$ will be invariant with respect to unknown input

$$\bar{e}^x_{k+1} = \bar{F}\,\bar{e}^x_k, \quad \bar{e}^u_k = B^{+}\left(Q\bar{F} - AQ\right)\bar{e}^x_k \qquad (9)$$

that grounds the proposed blind deconvolution problem solution.

Using the special form of mixing system model (1)

$$\begin{aligned} &A = \begin{pmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{pmatrix}, \\ &C = \begin{pmatrix} I_q & 0_{n-q\times q} \end{pmatrix}, B = \begin{pmatrix} B_1 \\ B_2 \end{pmatrix}\begin{matrix} q \\ n-q \end{matrix}, \end{aligned} \qquad (10)$$

which may be obtained by nonsingular state-space transformation, and concretely define the matrices P, Q choice, one can admit

$$\begin{pmatrix} P & Q \end{pmatrix} = \begin{pmatrix} P_1 & Q_1 \\ P_2 & Q_2 \end{pmatrix}\begin{matrix} q \\ n-q \end{matrix}, P_1 = I_q, Q_1 = 0_{q\times n-q}. \qquad (11)$$

In such a case, for any Q_2 such that $\det Q_2 \neq 0$, aggregating matrix may be found in the form $R = Q_2^{-1}\left(-P_2 \quad I_{n-q}\right)$, and consequently the UIO matrices for mixing system representation (10) are the following:

$$\begin{aligned}
&\overline{F} = R\Pi AQ = Q_2^{-1}\left(\tilde{A}_{22} - P_2\Omega_{B_1}A_{12}\right)Q_2, \\
&\tilde{A}_{22} = A_{22} - B_2B_1^{+}A_{12}, \quad \tilde{A}_{12} = \Omega_{B_1}A_{12} \\
&\Omega_{B_1} = I_q - B_1B_1^{+}.
\end{aligned} \tag{12}$$

Thus, in fact, the nonsingular matrix Q_2 specifies the similarity transformation and doesn't change the A^I spectrum, which as follows from (12), completely determined by only arbitrary tuning matrix P_2. The last may be choused by any type of pole placement method, and the problem will be solvable if matrix pair $(\tilde{A}_{22},\ \tilde{A}_{12})$ is observable. In particular, in such a way the designed inverse model may be stabilized for nonminimum-phase mixing system.

IV. Inverse model regularization

The observability condition is obviously violated in the typical case, when $q = p$. At that $\Omega_{B_1} = 0$, and $\overline{F}$ doesn't depend from. P_2 In such a case, for the tuning properties guarantee, it is expediently to use so-called "regularized" UIO [7], which ensure the approximate observer invariance with respect to unknown input signal.

$$\left\|RB - HCB\right\|^2 + \varepsilon\left\|H\right\|^2 \to \min_{H}, \tag{13}$$

where $\varepsilon > 0$ - regularization parameter. Using (5), (9) one can obtain the regularized solution of inverse model synthesis equations as follows:

$$\begin{aligned}
&\overline{F}(\varepsilon) = R\Pi(\varepsilon)AQ, \\
&\overline{G}(\varepsilon) = R\Pi(\varepsilon)A\left(\overline{H}(\varepsilon) + P\Omega(\varepsilon)\right), \\
&\overline{H}(\varepsilon) = RBS^{+}(\varepsilon),
\end{aligned} \tag{14}$$

where $\Pi(\varepsilon) = I_n - H(\varepsilon)C,\ \Omega_{B_1}(\varepsilon) = I_q - B_1B_1^{+}(\varepsilon)$. and the regularized projection matrices $\Pi(\varepsilon)$ and $\Omega_{B_1}(\varepsilon)$ has the following form:

$$\Pi(\varepsilon) = \begin{pmatrix} \Omega_{B_1}(\varepsilon) & 0_{q,n-q} \\ -B_2B_1^{+}(\varepsilon) & I_{n-q} \end{pmatrix},$$

$$\begin{aligned}
&\Omega_{B_1}(\varepsilon) = I_q - B_1B_1^{+}(\varepsilon), \\
&B_1^{+}(\varepsilon) = B_1^{\mathrm{T}}\left(\varepsilon I_q + B_1B_1^{\mathrm{T}}\right)^{-1}.
\end{aligned} \tag{15}$$

Matrix $S^{+}(\varepsilon) = S^{\mathrm{T}}\left(\varepsilon I_q + SS^{\mathrm{T}}\right)^{-1}$ may be considered as a regularized inverse of matrix S, at that $S^{+}(\varepsilon)\big|_{\varepsilon=0} = S^{-1}$ and $S^{+}(\varepsilon)\big|_{\varepsilon=\infty} = 0$.

As a result, dynamics matrix $\overline{F}(\varepsilon)$ of regularized reduced-order inverse model define as:

$$\begin{aligned}
&\overline{F}(\varepsilon) = \tilde{A}_{22}(\varepsilon) - P_2\tilde{A}_{12}(\varepsilon), \\
&\tilde{A}_{22}(\varepsilon) = A_{22} - B_2B_1^{+}(\varepsilon)A_{12}, \\
&\tilde{A}_{12}(\varepsilon) = \Omega_{B_1}(\varepsilon)A_{12},
\end{aligned} \tag{16}$$

Finally, taking into account, that $\Omega_{B_1}(\varepsilon) = \varepsilon\left(\varepsilon I_q + B_1B_1^{\mathrm{T}}\right)^{-1}$, from (14), (15) follows the reduced-order regularized inverse model equations:

$$\begin{aligned}
&\overline{x}_{k+1} = \overline{F}(\varepsilon)\overline{x}_k + \overline{L}(\varepsilon)y_k + \overline{H}(\varepsilon)y_{k+1}, \\
&\overline{u}_k = \overline{B}(\varepsilon)\left[y_{k+1} - CAQ\overline{x}_k - CAPy_k\right],
\end{aligned} \tag{17}$$

where

$$\begin{aligned}
&\overline{L}(\varepsilon) = R\Pi(\varepsilon)AP,\ \overline{H}(\varepsilon) = RBS^{+}(\varepsilon), \\
&\overline{B}(\varepsilon) = B^{+}\left(H(\varepsilon) + P\Omega(\varepsilon)\right), \\
&\Omega(\varepsilon) = I_q - SS^{+}(\varepsilon).
\end{aligned} \tag{18}$$

Since the following equalities take place

$$\Pi(\varepsilon)B = B\left(I_n - S^{+}(\varepsilon)S\right) = \varepsilon B\left(\varepsilon I_n + S^{\mathrm{T}}S\right)^{-1}, \tag{19}$$

the estimation error equations are:

$$\begin{aligned}
&\overline{e}_{k+1}^{x} = \overline{F}(\varepsilon)e_k^{x} + \varepsilon RB\left(\varepsilon I_n + S^{\mathrm{T}}S\right)^{-1}u_k, \\
&\overline{e}_k^{u} = B^{+}\left(Q\overline{F}(\varepsilon) - AQ\right)\overline{e}_k^{x} + \\
&+ \varepsilon B^{+}QRB\left(\varepsilon I_n + S^{\mathrm{T}}S\right)^{-1}u_k.
\end{aligned} \tag{20}$$

It is obvious, that under $q = m$, regularized projection matrix $\Omega_{B_1}(\varepsilon) \neq 0$ for any $\varepsilon > 0$ and inverse model design problem become solvable, if pair matrix $(\tilde{A}_{22}(\varepsilon), \tilde{A}_{11}(\varepsilon))$ is observable. The regularization parameter ε is selected based on trade-off between the desired observer dynamic properties, degree of stability in particular, and value of additional dynamic error component, proportional to the input signal, caused by inverse model regularization.

V. SOURCE SIGNAL IDENTIFICATION

Consider the source signal model as a superposition of single harmonics with arbitrary frequencies:

$$v_k = \sum_{j=0}^{m-1} \left[a_j \cos(\omega_j k) + b_j (\sin \omega_j k)\right] + \xi_k , \quad (21)$$

where v_k - some scalar component of source vector signal u_k at instant k, m - number of harmonics with frequencies $0 < \omega_j = 2\pi f_j T_0 < \pi$, T_0 - sampling period, ξ_k - random zero mean measurement noise.

Using z - transform, the model (21) may be presented in the form:

$$\prod_{j=0}^{m-1} \left[1 - 2\cos(\omega_j z^{-1}) + z^{-2}\right] v_k = \zeta_k . \quad (22)$$

Realizing the inverse transition in time domain, the equation (22) may be represent in the linear auto-regression form:

$$\begin{aligned} v_k &= \sum_{j=0}^{m-1} \beta_j \left(v_{k+j-m} + v_{k-j-m}\right) - v_{k-2m} + \zeta_k = \\ &= \beta^{\mathrm{T}} v(k,m) - v_{k-2m} + \zeta_k , \end{aligned} \quad (23)$$

where $v(k,m) = (2v_{k-m}, v_{k-m+1} + v_{k-m-1}, \ldots, v_{k-1} + v_{k-2m+1})^{\mathrm{T}}$ is the signal "prehistory" vector, $\beta^{\mathrm{T}} = (\beta_0, \beta_1, \ldots, \beta_{m-1})$ - model parameters.

Using the quadratic identification criterion

$$J = \sum_{k=2m}^{N-1} \left[v_k + v_{k-2m} - \beta^{\mathrm{T}} v(k,m)\right]^2 , \quad (24)$$

one can obtain the recurrent algorithm of source signal model identification:

$$\begin{aligned} \hat{\beta}_k &= \hat{\beta}_{k-1} + [v_k + v_{k-2m} - \\ &\qquad \beta^{\mathrm{T}}_{k-1} v(k,m)] v(k,m) r_k^{-1}, \\ r_k &= \gamma_k r_{k-1} + \|v(k,m)\|^2, \quad 0 < \gamma_k < 1, \end{aligned} \quad (25)$$

where tuning parameter γ may be used for trade-off adjusting between tracking and flittering properties of the algorithm (25).

Frequencies ω_j are connected with parameters β_j by the following equation:

$$\beta_0 + \sum_{j=1}^{m-1} \beta_j \cos(j\omega) = \cos(m\omega). \quad (26)$$

Consequently, an optimal one step forecast of source signal obtained by current estimates may be obtained as:

$$\hat{v}_i = \hat{\beta}^{\mathrm{T}}_{i-1} v(i,m) + v_{i-2m} . \quad (27)$$

Optimal prediction of wave signal for p steps $\hat{v}_{k+p}$ may be obtained in the similar manner using the one step prediction (27) :

$$\hat{v}_{k+p} = \hat{\beta}^{\mathrm{T}}_k \hat{v}(k+p,m) - v_{k+p-2m}, \; p \ge 1, \quad (28)$$

where elements v_i, $k \le i \le k+p-1$ in vector $\hat{v}(k+p,m)$ are replaced by their predicted values found in accordance with (27).

VI. CONCLUSIONS

The proposed method for blind deconvolution and source separation based on designed mixing system inverse model ensures the possibility of arbitrary unknown input signal real time estimation without necessity of essential a priory information about the signals. The advantage of suggested method is in the fact that the designed inverse model has the desired dynamic properties and may be stabilized for nonminimum-phase mixing system. The proposed approach also may be used for input signals identification and prediction as well as deconvolution inverse model stochastic optimization in the presence of random noise.

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